"Current-carrying states in superconductor/insulator and superconductor/semiconductor superlattices in the mesoscopic regime".

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We discuss some of the basic theoretical aspects of current-carrying states in superconducting superlattices with tunnel barriers in the mesoscopic regime, when $p_0^{-1} \ll a \ll \xi_0$ (a is the superconducting layer thickness, p_0 is the Fermi momentum, ξ_0 is the BCS coherence length, $\hbar = 1$). We establish the necessary conditions for the observation of the classical Josephson effect (with sinusoidal current-phase dependence) and derive self-consistent analytical expressions for the critical Josephson current. These expressions are proportional to the small factor a/ξ_0 and have unusual temperature dependence as compared with the single-junction case. For certain parameter values, the superconducting gap exhibits an exponential decrease due to pair-breaking effect of the supercurrent. The supercurrent can completely destroy the superconductivity of the system above a certain characteristic temperature T^*. In this paper, we also study the effect of intrabarrier exchange interactions. We show that this effect is strongly enhanced compared with the single-junction case and can manifest itself in an exponential decrease of the critical temperature.

PACS numbers: 74.50.+r, 70.80.Dm

I. INTRODUCTION

During the last years there have been major improvements in fabrication of high-quality vertically stacked superconducting periodic multilayers (superlattices) with Josephson coupling through insulating [1] and semiconducting [2] barriers. These devices are promising candidates for a variety of applications in microelectronics as, for example, high frequency oscillators, mixers or fast switches. [3–5] On the other hand, artificial Josephson coupled superlattices may serve [6,7] to model the properties of naturally layered high-T_c superconductors exhibiting the intrinsic Josephson effect. [8]

It is well-known [9] that the principal physical characteristic of any weakly-coupled superconducting system is the critical Josephson current, j_c. Unfortunately, the problem of self-consistent microscopic calculation of j_c in weakly-coupled superconducting superlattices poses serious mathematical difficulties and was first addressed only recently. [10,11] However, the very first theoretical results [10,11] revealed dramatic differences with respect to single-junction behavior. Whereas in the limit of thick superconducting (S) layers $a \gg \xi_0$ (a for the S-layer thickness, ξ_0 for the BCS coherence length) the description of the Josephson current in superlattices with tunnel barriers converges with that in single junctions, in the opposite *mesoscopic* regime $p_0^{-1} \ll a \ll \xi_0$ (p_0 is the Fermi momentum, $\hbar = 1$) drastic qualitative deviations from a single-junction case were found: a strong reduction of j_c due to the factor a/ξ_0, unusual temperature dependence, and the suppression of the gap parameter in the S-layers by the supercurrent. It should be emphasized that these subtle physical effects, arising due to nonlocality of j_c, are not seized by simple phenomenological models of layered superconductors, like, for example, the wide-spread Lawrence-Doniach model, [12] that start with the local description from the very beginning.

In this paper, we discuss the newly predicted nonlocal effects in mesoscopic superlattices with tunnel barriers in more detail. In Section II, we present a brief mathematical formulation of the problem. In Section III, we consider superlattices with finite-size nonmagnetic insulating and semiconducting barriers. In Section IV, we investigate the influence of intrabarrier exchange interactions. Finally, in Section V, we present a summary of the results and make some concluding remarks.

II. MATHEMATICAL FORMALISM

Let us first formulate precisely the problem we are going to consider. Throughout the paper, we study infinite periodic in the x-direction superconducting (s-wave) systems with interlayer Josephson coupling in the clean limit

and in the absence of external magnetic fields. Complete structural homogeneity in the yz-plane is implied, though the transverse dimensions of the systems are taken to be small compared to the London penetration depth in order to discard the influence of self-induced fields and reduce the problem to effectively one-dimensional. The S-layers and the barriers occupy the regions $S_n = [-a - d/2 + nc, -d/2 + nc]$ and $B_n = [-d/2 + nc, d/2 + nc]$, respectively ($c = a + d$ is the period, and n is an integer). Under these conditions, the problem is described mathematically by the Gor'kov equations (Fourier-transformed in y, z):

$$\left\{ i\omega + \left[E_F t^2 + \frac{1}{2m}\frac{d^2}{dx^2} - \hat{U}_B(x) \right] \tau_3 + \frac{i}{2}(\tau_1 + i\tau_2)\sigma_2 \Delta(x) \right.$$

$$\left. - \frac{i}{2}(\tau_1 - i\tau_2)\sigma_2 \Delta^*(x) \right\} \begin{bmatrix} \hat{G}_\omega(x, x'; t) \\ \hat{F}_\omega(x, x'; t) \end{bmatrix} = \begin{bmatrix} \delta(x - x') \\ 0 \end{bmatrix}, \tag{1}$$

$$\Delta^*(x) = -\frac{i}{2}\left| g(x) \right| \pi N(0) v_0 T \sum_\omega \int_0^1 dt t \,\mathrm{tr} \left\langle \hat{F}_\omega(x, x; t)\sigma_2 \right\rangle, \tag{2}$$

$$g(x) = -|g| \sum_n \delta_{S_n}(x),$$

$$\delta_\Omega(x) = \{1, \text{ for } x \in \Omega; 0, \text{ for } x \notin \Omega\}.$$

Here, $\hbar = 1$, τ_i $(i = 1, 2, 3)$ are the Pauli matrices in the Gor'kov-Nambu space, σ_i $(i = 1, 2, 3)$ are the Pauli matrices in the spin space, $\omega = \pi T(2n + 1)$ (n is an integer), E_F is the Fermi energy, $N(0) = mp_0/2\pi^2$ is the one-spin density of states at the Fermi level ($p_0 = mv_0$ being the Fermi momentum), g is the electron-electron coupling constant, $t \equiv \cos\theta$ is the cosine of the angle of incidence at the interface. The barrier potential is given by $\hat{U}_B(x)$, where the accent ($\hat{}$) denotes a non-trivial matrix structure in the spin space. [In the absence of intrabarrier exchange interactions $\hat{U}_{B\alpha\beta}(x) = U_B(x)\delta_{\alpha\beta}$.] The functions $\hat{G}_\omega$, $\hat{F}_\omega$ are 2x2 matrices in the spin space. In the self-consistency equation (2), the trace is taken over the spin indices, and $\langle \ldots \rangle$ denotes spatial averaging over atomic-scale oscillations. (We confine ourselves to the limit $p_0^{-1} << a$). Because of the periodicity, the pair-potential obeys the relation $\Delta(x + nc) = \Delta(x)\exp(in\phi)$. The functions $\hat{G}_\omega$, $\hat{F}_\omega$ and their first derivatives $\hat{G}'_\omega$, $\hat{F}'_\omega$ are subject to the usual continuity conditions at the interfaces $x = \pm d/2 + nc$.

Our main interest is focused on the supercurrent density. For this quantity, we employ the integral representation [11]

$$j \equiv j(x \in B_0)$$

$$= -2\pi e v_0 N(0) T \sum_\omega \int_0^1 dtt \int_{-\infty}^{-d/2} dx_1 \int_{d/2}^{\infty} dx_2 \left[\left\langle \mathrm{tr}\left[\hat{G}^n_\omega(x_1, x_2; t)\sigma_2 \hat{G}^t_{-\omega}(x_2, x_1; t)\sigma_2 \right] \right\rangle \right.$$

$$\left. \times \, \mathrm{Im}\left[\Delta(x_1)\Delta^*(x_2)\right] \right], \tag{3}$$

where $\hat{G}^n_\omega$ is the Green's function of the system in the normal state, and the upper index $(^t)$ means transposition in the spin space. An obvious advantage of this representation is the proportionality of the integrand to the product of the Green's functions with $x_1 \in S_m$, $x_2 \in S_n$, where $m \neq n$, identically equal to zero in the absence of weak coupling: To evaluate j_c in first order in the tunneling probability, D, we must take Δ in zero order and substitute the expressions for $\hat{G}^n_\omega$, $\hat{G}^t_{-\omega}$ in first order in $\sqrt{D}$. In this manner, we can circumvent a very difficult problem of finding the full Green's function and calculate only physically relevant elements of the latter, entering Eq. (3), on the basis of a perturbation expansion for Eqs. (1), (2) and the boundary conditions.

III. SUPERCONDUCTOR/INSULATOR (S/I) AND SUPERCONDUCTOR/SEMICONDUCTOR (S/SEM) SUPERLATTICES WITH NONMAGNETIC BARRIERS

We begin by considering the case of a periodic structure with a finite-size nonmagnetic repulsive barrier of the form

$$\hat{U}_{B\alpha\beta}(x) = U_0 \delta_{\alpha\beta} \sum_n \delta_{B_n}(x), \; U_0 > 0, \tag{4}$$

which is typical of superconductor/semiconductor (S/Sem) multilayers. For the potential (4), the dependence of functions $\hat{G}_\omega$, $\hat{F}_\omega$ on the spin indices is trivial,

$$\left[\hat{G}_\omega\right]_{\alpha\beta} = G_\omega \delta_{a\beta}, \;\; \left[\hat{F}_\omega\right]_{\alpha\beta} = F_\omega i\sigma_{2\alpha\beta},$$

and can be suppressed in what follows.

As we are not concerned with the Green's functions with coordinates inside the barriers, we shall consider only $G_\omega(x \in S_n, x' \in S_m; t)$, $F_\omega(x \in S_n, x' \in S_m; t)$. To derive the boundary conditions for these functions, we first solve (1) for $G_\omega(x \in B_n, x' \in S_m; t)$ and $F_\omega(x \in B_n, x' \in S_m; t)$. Making use of the full set of the boundary conditions, we arrive at the required relations:

$$\begin{bmatrix} G_\omega \\ F_\omega \end{bmatrix} (nc + d/2, x') = \cosh\left(\lambda^{\mp}_B d\right) \begin{bmatrix} G_\omega \\ F_\omega \end{bmatrix} (nc - d/2, x')$$

$$+ \frac{\sinh\left(\lambda^{\mp}_B d\right)}{\lambda^{\mp}_B} \begin{bmatrix} G'_\omega \\ F'_\omega \end{bmatrix} (nc - d/2, x'), \tag{5}$$

$$\begin{bmatrix} G'_\omega \\ F'_\omega \end{bmatrix} (nc + d/2, x') = \cosh\left(\lambda_B^{\mp} d\right) \begin{bmatrix} G'_\omega \\ F'_\omega \end{bmatrix} (nc - d/2, x')$$

$$+ \lambda_B^{\mp} \sinh\left(\lambda_B^{\mp} d\right) \begin{bmatrix} G_\omega \\ F_\omega \end{bmatrix} (nc - d/2, x'), \tag{6}$$

where $\lambda_B^{\mp} = \sqrt{2m\left(U_0 - E_F t^2 \mp i\omega\right)}$. Equations (5), (6) form a closed system of exact boundary conditions for the functions $G_\omega(x \in S_n, x' \in S_m; t)$, $F_\omega(x \in S_n, x' \in S_m; t)$. In the limit $d \to +0$, $U_0 \to +\infty$, $dU_0 \equiv V =$ const, they reduce to the boundary conditions for a periodic delta-function potential.

Assuming $\lambda_B^{\mp} \approx \lambda_B \equiv \sqrt{2m\left(U_0 - E_F t^2\right)}$, we proceed to the limit of a low-transparency barrier $\lambda_B d >> 1$. We can now solve (1), (2) with the boundary conditions (5), (6) by means of perturbation theory, with $\exp(-\lambda_B d) \propto \sqrt{D}$ being the expansion parameter. As expected, in zero order, $\Delta^{(0)}(x) = \Delta_0(T) \sum_n \exp(in\phi)\delta_{S_n}(x)$ (Δ_0 is the gap in the bulk, ϕ is a phase shift at the interfaces), and only $G_\omega^{(0)}(x \in S_n, x' \in S_n; t)$ and $F_\omega^{(0)}(x \in S_n, x' \in S_n; t)$ are nonzero. The functions $G_\omega(x \in S_n, x' \in S_m; t)$, $F_\omega(x \in S_n, x' \in S_m; t)$ with $|n - m| > 1$ are of order > 1 in $\sqrt{D}$ and should be neglected. The first-order approximation to $G_\omega(x \in S_1, x' \in S_0; t)$, entering (1), is given by

$$G_\omega^{(1)}(x \in S_1, x' \in S_0; t) =$$

$$= -\frac{m\lambda_B e^{-\lambda_B d}}{\Omega^2} \left\{ \frac{(\Omega + \omega)^2 + \Delta_0^2 e^{i\phi}}{\lambda_+^2 + \lambda_B^2} \frac{\sin\left[\lambda_+(a + d/2 - x + \alpha_+)\right] \sin\left[\lambda_+(a + d/2 + x' + \alpha_+)\right]}{\sin^2\left[\lambda_+(a + \beta_+)\right]} \right.$$

$$+ \frac{\Delta_0^2\left(1 - e^{i\phi}\right)}{\lambda_+^2 + \lambda_B^2} \sqrt{\frac{\lambda_+^2 + \lambda_B^2}{\lambda_-^2 + \lambda_B^2}} \frac{\sin\left[\lambda_+(a + d/2 - x + \alpha_+)\right] \sin\left[\lambda_-(a + d/2 + x' + \alpha_-)\right]}{\sin\left[\lambda_+(a + \beta_+)\right] \sin\left[\lambda_-(a + \beta_-)\right]}$$

$$+ \frac{\Delta_0^2\left(1 - e^{i\phi}\right)}{\lambda_-^2 + \lambda_B^2} \sqrt{\frac{\lambda_-^2 + \lambda_B^2}{\lambda_+^2 + \lambda_B^2}} \frac{\sin\left[\lambda_-(a + d/2 - x + \alpha_-)\right] \sin\left[\lambda_+(a + d/2 + x' + \alpha_+)\right]}{\sin\left[\lambda_+(a + \beta_+)\right] \sin\left[\lambda_-(a + \beta_-)\right]}$$

$$\left. + \frac{(\Omega - \omega)^2 + \Delta_0^2 e^{i\phi}}{\lambda_-^2 + \lambda_B^2} \frac{\sin\left[\lambda_-(a + d/2 - x + \alpha_-)\right] \sin\left[\lambda_-(a + d/2 + x' + \alpha_-)\right]}{\sin^2\left[\lambda_-(a + \beta_-)\right]} \right\}, \tag{7}$$

where $\Omega = \sqrt{\omega^2 + \Delta_0^2}$, $\lambda_\pm = \pm\sqrt{2m\left(E_F t^2 \pm i\Omega\right)}$, $\alpha_\pm = \lambda_\pm^{-1} \arcsin\left[\lambda_\pm\left(\lambda_\pm^2 + \lambda_B^2\right)^{-1/2}\right]$, and $\beta_\pm = \lambda_\pm^{-1} \arcsin\left[2\lambda_\pm\lambda_B\left(\lambda_\pm^2 + \lambda_B^2\right)^{-1/2}\right]$. The function $G_\omega^{(1)}(x \in S_0, x' \in S_1; t)$, also entering (1), is obtained from (7) via the substitution $x \leftrightarrow x'$, $\phi \to -\phi$. The normal-state function $G_\omega^{n(1)}(x \in S_1, x' \in S_0; t)$ is a limiting case of (7) for $\Delta_0 = 0$. Note that the spatial dependence of (7) clearly indicates that in first order in D only two adjacent S-layers contribute to the supercurrent (3).

Now one can benefit from the quasiclassical approximation $\lambda_\pm \approx \pm p_0|t| + i\Omega/v_0|t|$ $[|t| >> (T_{c0}/E_F)^{1/2}]$. Inserting $\Delta^{(0)}$ and quasiclassical expressions for $G^{n(1)}_\omega$, $G^{(1)}_\omega$ into (3), carrying out small-scale averaging and performing spatial integration finally yields

$$j = \frac{\pi}{2} e v_0 N(0) \Delta_0^2 T \int_0^1 dt t D(t) \sum_\omega \frac{\tanh \frac{2a\sqrt{\omega^2+\Delta_0^2}}{v_0 t}}{\omega^2 + \Delta_0^2} \sin\phi, \tag{8}$$

$$D(t) = \frac{16 E_F t^2 \left(U_0 - E_F t^2\right)}{U_0^2} \exp\left[-2d\sqrt{2m\left(U_0 - E_F t^2\right)}\right].$$

Equation (8) is the desired analytical expression for the dc Josephson current in clean S/I- and S/Sem-multilayers, valid for any $p_0^{-1} << a \leq \infty$ and arbitrary temperatures. [Concerning certain limitations of the application of Eq. (8) to *mesoscopic* superlattices, see the discussion of Eq. (10) below.] We observe that (8) does not depend on concrete features of our model and should also hold for semiconducting barriers with internal structures of the type considered by Aslamazov and Fistul in the case of a single SSemS-junction [13]. Equation (8) is independent of the period c. This is a direct consequence of the already-mentioned adjacent-layer coupling in first order in D. The period may enter higher-order corrections, when the effect of subgap current-carrying Bloch states [14] comes into play.

As expected, in the limit $a >> \xi_0$, (8) goes over into the Ambegaokar-Baratoff relation [15]

$$j = \frac{\pi e v_0 N(0)}{2} \int_0^1 dt t D(t) \Delta_0(T) \tanh\frac{\Delta_0(T)}{2T} \sin\phi. \tag{9}$$

However, the most interesting is the *mesoscopic regime*, when $p_0^{-1} << a << \xi_0$. In this regime, using the gap equation for a bulk superconductor, we arrive at the fundamental result

$$j = \frac{1}{2\pi} \frac{e v_0 N(0) \Delta_0^2(T)}{T_{c0}} \frac{a}{\xi_0} \frac{\int_0^1 dt D(t)}{|g|\, N(0)} \sin\phi, \tag{10}$$

where $\xi_0 \equiv v_0/2\pi T_{c0}$.

First we note that the form of expression (10) implies that in the mesoscopic regime the true expansion parameter for the Josephson current is the ratio $D/|g|\, N(0) \ll 1$ instead of $D \ll 1$ in the Ambegaokar-Baratoff regime. [This conjecture is verified below. See the discussion of relation (15).] Taking account of the fact that in most low-T_c superconductors $|g|\, N(0)$ is typically <0.3, we infer that the classical Josephson effects in mesoscopic superlattices can be observed only under severe restrictions on the upper bound for D.

Three other remarkable features are to be noted with regard to (10). (i) A strong reduction of j_c due to the emergence of the additional small factor, a/ξ_0. (ii) The temperature dependence is determined solely by the factor

$\Delta_0^2(T)$ in the whole temperature range. (iii) The occurrence of $\int_0^1 dtD(t)$ instead of $\int_0^1 dttD(t)$ in the Ambegaokar-Baratoff regime. These results is a manifestation of the nonlocality of the supercurrent in its extreme: While the product of two Green's functions in the integrand of (3) decays at distances on the order of ξ_0, the actual range of spatial integration is restricted by two adjacent S-layers only. Thus, for instance, at $T = 0$ we can rewrite (10) as

$$j = ev_0 N(0)\Delta_0(0)\frac{\int_0^1 dttD(t)P(t)}{|g|\,N(0)}\sin\phi,$$

where $P(t) = a/v_0 t\Delta_0^{-1}(0)$ is the quasiclassical probability [16] of finding an unscattered electron with the x-component of the velocity $v_0 t$ within one S-layer during the characteristic time $\Delta_0^{-1}(0)$. Finally, one must bear in mind that (10) has been derived in the clean limit. Considerable changes may occur in the dirty limit $l << \xi_0$ (l is the electron mean free path), when the fall-off length of the integrand in (3) is of the order of $\sqrt{\xi_0 l}$. Actually, this limiting situation asks for further investigation.

Now we shall study the effect of suppression of the gap parameter by the supercurrent in the mesoscopic regime, and establish the exact domain of validity of Eq. (10), obtained by perturbation methods. To this end, we should consider the linearized self-consistency equation (2). For the sake of simplicity, we restrict ourselves to the case of thin ($d << \min\{a, \xi_0\}$) insulating repulsive barriers. For such barriers, the sought equation, expanded to first order in $D << 1$, reads:

$$\Delta(x \in S_0) = \frac{\pi N(0)\,|g|}{v_0}\left[\int_{-a+0}^{-0} dx'\Delta(x')T\sum_\omega \int_0^1 \frac{dt}{t}\left\{\exp\left[-\frac{2|\omega|}{v_0 t}\,|x-x'|\right]\right.\right.$$

$$\left.[1-D(t)]\,\frac{\exp\left[-\frac{2|\omega|a}{v_0 t}\right]\cosh\left[\frac{2|\omega|}{v_0 t}\,(x-x')\right]+\cosh\left[\frac{2|\omega|}{v_0 t}\,(x+x'+a)\right]}{\sinh\frac{2|\omega|a}{v_0 t}}\right\}$$

$$+\int_{+0}^{a-0} dx'\Delta(x')T\sum_\omega \int_0^1 \frac{dt}{t}D(t)\left\{\frac{\exp\left[-\frac{2|\omega|}{v_0 t}\,(x'-x-a)\right]}{2\cosh\frac{2|\omega|a}{v_0 t}}\right.$$

$$\left.+\frac{\exp\left[-\frac{2|\omega|a}{v_0 t}\right]\cosh\left[\frac{2|\omega|}{v_0 t}\,(x-x'+a)\right]+\cosh\left[\frac{2|\omega|}{v_0 t}\,(x+x')\right]}{\sinh\frac{4|\omega|a}{v_0 t}}\right\}$$

$$+\int_{-2a+0}^{-a-0} dx'\Delta(x')T\sum_\omega \int_0^1 \frac{dt}{t}D(t)\left\{\frac{\exp\left[-\frac{2|\omega|}{v_0 t}\,(x-x'-a)\right]}{2\cosh\frac{2|\omega|a}{v_0 t}}\right.$$

$$\left.\left.+\frac{\exp\left[-\frac{2|\omega|a}{v_0 t}\right]\cosh\left[\frac{2|\omega|}{v_0 t}\,(x-x'-a)\right]+\cosh\left[\frac{2|\omega|}{v_0 t}\,(x+x'-2a)\right]}{\sinh\frac{4|\omega|a}{v_0 t}}\right\}\right], \tag{11}$$

with a cutoff at the Debye frequency, ω_D, in the sum over ω implied. The restriction on spatial integration by two adjacent S-layers is again a result of the first-order approximation. For $a >> \xi_0$, (11) goes over into the very familiar equation for a single SIS-junction, [17] describing spatial dependence of Δ near T_{c0} in the vicinity of the barrier. The analysis of (11) in the limit $p_0^{-1} << a << \xi_0$ shows that in the current-carrying state it has non-trivial solutions for $T < T_{c0}$. These solutions are given by a complex Δ, constant in each S-layer with a phase shift ϕ_c at the interfaces. (Physically, but for the phase jumps ϕ no spatial variations can occur in the small-scale-averaged Δ over distances less than ξ_0.) Thus, substituting $\Delta(x) = \Delta(T) \sum_{n=-1}^{1} \exp(in\phi_c)\delta_{S_n}(x)$ into Eq. (11) yields

$$|\phi_c| = \arccos\left[1 + \frac{|g|N(0)}{\int_0^1 dtD(t)} \ln\frac{T}{T_{c0}}\right], \tag{12}$$

where $T^* \leq T \leq T_{c0}$, with

$$T^* = T_{c0}\exp\left[-\frac{1}{|g|N(0)}\int_0^1 dtD(t)\right]. \tag{13}$$

The physical meaning of relations (12), (13) is unraveled by the fact that in the temperature range $T^* \leq T \leq T_{c0}$ the superconductivity of the S-layers completely vanish due to depairing effect of the supercurrent, when the phase difference reaches the critical value $|\phi| = |\phi_c|$. In this sense, the critical phase difference $|\phi_c|$ in mesoscopic superlattices plays the same role as the critical superfluid velocity [17,18] $v_c \sim \Delta(T)/p_0$ in thin superconducting wires.

At temperatures $T < T^*$, the Josephson current cannot completely destroy the superconductivity of S-layers, but still one has to face strong suppression of the gap parameter, unless the condition $D/\left|g\right|N(0) \ll 1$ is fulfilled. Thus, at $T = 0$ we have

$$|\Delta(0)| = \Delta_0(0)\exp\left[-\frac{1}{|g|N(0)}\int_0^1 dtD(t)\left(1 - \cos\phi\right)\right], \tag{14}$$

where $\Delta_0(0) = 2\omega_D \exp\left[-\frac{1}{|g|N(0)}\right]$ is the bulk gap at $T = 0$. In this case, the influence of the Josephson current can even be regarded as effective weakening of the electron-electron coupling constant:

$$|g| \to |g|\left[1 - \int_0^1 dtD(t)\left(1 - \cos\phi\right)\right].$$

[Analogous renormalization of $|g|$ owing to pair-breaking is known for proximity-effect S/N-bilayers in the so-called Cooper limit. [19]] As a consequence, for $D \sim \left|g\right|N(0)$, we expect non-sinusoidal current-phase dependence in the whole temperature range.

On the contrary, for

$$\int_0^1 dtD(t) << |g|N(0), \tag{15}$$

equation (10) gives a fully self-consistent description of the Josephson current at $T < T^*$, whereas the situation in the temperature range $T^* \leq T \leq T_{c0}$ yet needs clarification.

IV. THE EFFECT OF INTRABARRIER EXCHANGE INTERACTION

To investigate the effect of intrabarrier exchange interactions, we again turn to the simplest case of thin insulating barriers with $d << \min\{a, \xi_0\}$. In this case, the barrier potential can be modeled by [20]

$$\hat{U}_B(x) = (V + J\sigma_3) \sum_{n=-\infty}^{+\infty} \delta(x - na), \;\; V > 0, \tag{16}$$

where V and J are the nonexchange and exchange parts, respectively.

In the limit of a low barrier transmission, when $V \gg |J| \gg v_0$, we can employ the perturbation procedure described in Sec. III, obtaining

$$\hat{G}^{(1)}_\omega(x \in S_1, x' \in S_0; t) =$$

$$= -\frac{1}{4\Omega^2}\frac{V - J\sigma_3}{V^2}\left\{\left[(\Omega+\omega)^2 + \Delta_0^2 e^{i\phi}\right]\frac{\sin[\lambda_+(a-x)]\sin[\lambda_+(a+x')]}{\sin^2\lambda_+ a}\right.$$

$$+\Delta_0^2\left(1 - e^{i\phi}\right)\frac{\sin[\lambda_+(a-x)]\sin[\lambda_-(a+x')] + \sin[\lambda_-(a-x)]\sin[\lambda_+(a+x')]}{\sin\lambda_+ a\sin\lambda_- a}$$

$$\left. + \left[(\Omega-\omega)^2 + \Delta_0^2 e^{i\phi}\right]\frac{\sin[\lambda_-(a-x)]\sin[\lambda_-(a+x')]}{\sin^2\lambda_- a}\right\}. \tag{17}$$

[Compare with Eq. (7)]. The Green's function in the normal state, $\hat{G}^{n(1)}_\omega$, can be obtained from (17) by taking the limit $\Delta_0 \to 0$.

Substituting $\Delta^{(0)}(x) = \Delta_0(T)\sum_{n=-1}^{1}\exp(in\phi)\delta_{S_n}(x)$ and quasiclassical approximation for $\hat{G}^{n(1)}_\omega$, $\hat{G}^{(1)}_\omega$ into (3) with $d = 0$, we get

$$j = \frac{\pi}{2} e v_0 N(0)\Delta_0^2 T \int_0^1 dtt\,[D(t) - 2D_S(t)] \sum_\omega \frac{\tanh\frac{2a\sqrt{\omega^2+\Delta_0^2}}{v_0 t}}{\omega^2 + \Delta_0^2}\sin\phi, \tag{18}$$

where

$$D(t) = \frac{v_0^2\left(V^2 + J^2\right)}{V^4}, \quad D_S(t) = \frac{v_0^2 J^2}{V^4}$$

are the total tunneling probability and the exchange part of the tunneling probability, respectively. In Eq. (18), the typical difference $D - 2D_S$ can be regarded [21,22] as the tunneling probability for a Cooper pair, with $2D_S$ being the probability of pair breaking. In the limit $D_S = 0$ ($J = 0$), equation (18) reduces to (8), as it should.

For $a \gg \xi_0$, equation (18) becomes [22]

$$j = \frac{\pi e v_0 N(0)}{2} \int_0^1 dtt \left[D(t) - 2D_S(t)\right] \Delta_0(T) \tanh \frac{\Delta_0(T)}{2T} \sin\phi. \tag{19}$$

In the opposite *mesoscopic* limit $p_0^{-1} \ll a \ll \xi_0$, we obtain

$$j = \frac{1}{2\pi} \frac{e v_0 N(0) \Delta_0^2(T)}{T_{c0}} \frac{a}{\xi_0} \frac{\int_0^1 dt \left[D(t) - 2D_S(t)\right]}{|g|\, N(0)} \sin\phi. \tag{20}$$

This equation implies that $(D - 2D_S) / |g|\, N(0) \ll 1$. As in the case of Eq. (10), to establish the temperature range of validity of Eq. (20), we should consider the corresponding linearized equation

$$\Delta(x \in S_0) = \frac{\pi N(0)\,|g|}{v_0} \left[\int_{-a+0}^{-0} dx' \Delta(x') T \sum_{\omega} \int_0^1 \frac{dt}{t} \left\{ \exp\left[-\frac{2|\omega|}{v_0 t}|x - x'|\right] \right.\right.$$

$$\left. [1 - D(t) - 2D_S(t)] \frac{\exp\left[-\frac{2|\omega|a}{v_0 t}\right] \cosh\left[\frac{2|\omega|}{v_0 t}(x - x')\right] + \cosh\left[\frac{2|\omega|}{v_0 t}(x + x' + a)\right]}{\sinh \frac{2|\omega|a}{v_0 t}} \right\}$$

$$+ \int_{+0}^{a-0} dx' \Delta(x') T \sum_{\omega} \int_0^1 \frac{dt}{t} [D(t) - 2D_S(t)] \left\{ \frac{\exp\left[-\frac{2|\omega|}{v_0 t}(x' - x - a)\right]}{2\cosh \frac{2|\omega|a}{v_0 t}} \right.$$

$$\left. + \frac{\exp\left[-\frac{2|\omega|a}{v_0 t}\right] \cosh\left[\frac{2|\omega|}{v_0 t}(x - x' + a)\right] + \cosh\left[\frac{2|\omega|}{v_0 t}(x + x')\right]}{\sinh \frac{4|\omega|a}{v_0 t}} \right\}$$

$$+ \int_{-2a+0}^{-a-0} dx' \Delta(x') T \sum_{\omega} \int_0^1 \frac{dt}{t} [D(t) - 2D_S(t)] \left\{ \frac{\exp\left[-\frac{2|\omega|}{v_0 t}(x - x' - a)\right]}{2\cosh \frac{2|\omega|a}{v_0 t}} \right.$$

$$\left.\left. + \frac{\exp\left[-\frac{2|\omega|a}{v_0 t}\right] \cosh\left[\frac{2|\omega|}{v_0 t}(x - x' - a)\right] + \cosh\left[\frac{2|\omega|}{v_0 t}(x + x' - 2a)\right]}{\sinh \frac{4|\omega|a}{v_0 t}} \right\} \right]. \tag{21}$$

In the limit $a >> \xi_0$, equation (21) goes over into the equation for a single junction with a tunnel magnetic barrier, [21] describing spatial dependence of Δ near T_{c0} in the vicinity of the barrier. For $D_S = 0$, equation (21) reduces to (11), as expected. But by contrast to Eq. (11), equation (21) has non-trivial solution at $T < T_{c0}$ even for real $\Delta(x) = \Delta(T) \sum_{n=-1}^{1} \delta_{S_n}(x)$ (i. e., in the absence of the supercurrent). These solutions determine the critical temperature T_c:

$$T_c = T_{c0} \exp\left[-\frac{4}{|g| N(0)} \int_0^1 dt D_S(t)\right]. \tag{22}$$

Expression (22) has a standard form of the critical temperature in the presence of non-ergodic pair-breaking interactions, typical of small-size (on the scale ξ_0) superconducting systems. [17] As we see, for $D_S \sim |g|\,N(0)$, the temperature shift due to intrabarrier exchange interactions in mesoscopic superlattices cannot be disregarded, whereas in the limit $a \gg \xi_0$, with $D_S \not= 0$, one would have [21] $T_c = T_{c0}$.

Substituting now $\Delta(x) = \Delta(T) \sum_{n=-1}^{1} \exp(in\phi_c)\delta_{S_n}(x)$, we determine the critical phase difference $|\phi_c|$:

$$|\phi_c| = \arccos\left[1 + \frac{|g|N(0)}{\int_0^1 dt\,[D(t) - 2D_S(t)]} \ln\frac{T}{T_c}\right], \tag{23}$$

where $T^* \leq T \leq T_c$, with T^* given by

$$T^* = T_c \exp\left[-\frac{1}{|g|N(0)} \int_0^1 dt\,[D(t) - 2D_S(t)]\right]. \tag{24}$$

At temperatures $T < T^*$, the superconductivity of the S-layers cannot be destroyed completely by the Josephson current, although the suppression of the gap parameter is not necessarily small. For example, at $T = 0$ the gap is given by

$$|\Delta(0)| = \Delta_0(0) \exp\left[-\frac{1}{|g|N(0)} \int_0^1 dt\,[D(t)\,(1 - \cos\phi) + 2D_S\,(1 + \cos\phi)]\right]. \tag{25}$$

Thus, combined pair-breaking effect of the intrabarrier exchange interactions and the Josephson current can be regarded as an effective renormalization of the BCS coupling constant:

$$|g| \to |g| \left[1 - \int_0^1 dt\,[D(t)\,(1 - \cos\phi) + 2D_S\,(1 + \cos\phi)]\right].$$

In the limit

$$\int_0^1 dt\,[D(t) - 2D_S(t)] \ll |g|N(0), \tag{26}$$

the exponential in Eq. (25) can be expanded into a power series, which proves the self-consistency of Eq. (20) for $T < T^*$.

V. SUMMARY AND CONCLUSIONS

Summarizing, we have discussed some of the basic theoretical aspects of current-carrying states in superconducting mesoscopic superlattices with tunnel barriers.

In particular, we have derived self-consistent analytical expressions for the Josephson current in these structures [Eqs. (10) and (20)], valid under conditions (15) and (26), respectively, at temperatures $T < T^*$, with T^* given

by (13) and (24). We have explained the peculiarities of Eqs. (10) and (20), i. e., a strong depression of j_c due to the factor a/ξ_0 and unusual temperature dependence given by $\Delta_0^2(T)$, in terms of nonlocality inherent to the theory of superconductivity. We have shown that in the temperature range $T^* \leq T < T_c$ the Josephson current can completely destroy the superconductivity of the S-layers at a certain critical phase difference $|\phi_c|$ [Eqs. (12) and (23)]. Our equations (14), (25) reveal an exponential decrease of the gap parameter at $T = 0$ due to the Josephson current, unless the conditions (15), (26) are fulfilled. Moreover, equations for the critical temperature (22) and the gap parameter at $T = 0$ (25) in the case of magnetic tunnel barriers clearly demonstrate that, by contrast to the single-junction case, the influence of intrabarrier exchange interactions cannot be completely disregarded in the mesoscopic superlattices too.

The above results may have important practical implications for superconducting low-T_c technology. For example, the predicted effect of a strong reduction of j_c in the mesoscopic regime establishes a quantitative limit on decreasing the S-layer thickness in vertically-stacked Josephson-junction arrays intended for superconducting microelectronic circuitry of high integration.

Finally, we cannot but mention some of unresolved problems. As we have already pointed out, the situation in the temperature range $T^* \leq T < T_c$ needs further clarification, as well as the effect of nonmagnetic impurities in the S-layers. Another interesting issue is the combination of the above discussed effects, resulting from spatial nonlocality, with anisotropic (d-wave) pairing now proposed for high-T_c superconductors. All this suggests that further theoretical and experimental studies of weakly-coupled mesoscopic superlattices would be highly desirable.

ACKNOWLEDGMENTS

The authors gratefully acknowledge stimulating discussions with Š. Beňačka, U. Gunsenheimer, A. N. Omel'yanchuk, S. Takács, and I. Vávra.

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